A Novel Method to Reduce Cogging Torque in Permanent Magnet Machines with Different Slot Opening Width

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Abstract —This paper proposed a novel method for reducing cogging torque in permanent magnet (PM) machines with different width of slot opening. In this method, the width of one slot opening is different from that of the others while all the tooth tips are of the same width. Firstly, the analytical expression of cogging torque accounting for different widths of slot openings was derived to analyze the detailed principle and influence of the proposed method on cogging torque qualitatively. Then analytical method was proposed to determine the optimal slot opening width to minimize cogging torque. At last, two prototype PM machines, i.e. a PMSM and a BLDC are designed and analyzed, respectively. The cogging torque calculated by FEA strongly verified the proposed methods.

I. INTRODUCTION

In slotted PM machines, cogging torque, which may affect the control accuracy and cause vibrations, is produced as a result of the interaction between permanent magnets and armature. Many studies have been carried out on the calculation, analysis and reduction of cogging torque [1]-[4]. From the view point of machine design, generally speaking, these methods can be classified into three groups: 1) Selecting a proper match of the number of slots and the number of poles; 2) Changing design parameters of the magnetic pole; 3) Changing design parameters of armature. This paper proposes a novel method for reducing cogging torque by changing the width of slot openings which can contribute to the third group.

II. NON-UNIFORM DISTRIBUTED SLOT OPENING WIDTH PROPOSED THIS PAPER

Normally, all the slot openings in PM machines are of the same width, as Fig.1 shows. The schematic of the method proposed in this paper is shown in Fig.2. In this method, the width of one slot opening is different from that of the others while all teeth tips are of the same width. All the widths of teeth tips in stator remain invariable before and after adopting this method.

As shown in Fig. 2, the number of slots of the machine is z. The width of tooth tip is θ_c. The width of one slot opening is θ_a, while that of the other z-1 slot opening is θ_b, thus

$$\theta_a + \theta_b(z-1) + \theta_c z = 2\pi \tag{1}$$

For convenient study, the ratio of θ_b to θ_a is defined as the slot opening width ratio k_s, so θ_a can be expressed as

$$\theta_a = \frac{2\pi - z\theta_c}{(z-1)k_s + 1} \tag{2}$$

It is obvious that, when k_s=1 is satisfied, all the slot openings are of the same width, thus the slot openings distribute uniformly. Once the width of tooth tip, i.e. θ_c is given, the width of slot opening can be directly determined by k_s. It can be seen that this method permits the designer to choose any suitable width of the teeth tips at willing to reduce cogging torque.

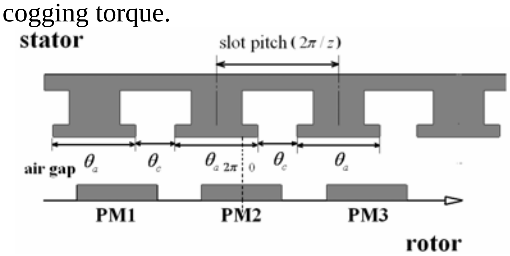


Fig. 1 Stator of PM machines with uniformly distributed slot opening width

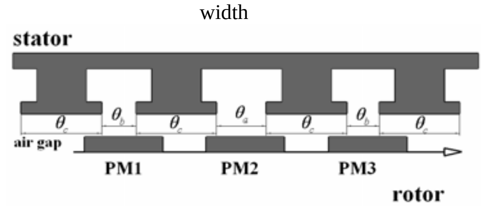


Fig. 2 Non-uniform distributed slot opening width proposed in this paper

III. ANALYTICAL MODEL OF COGGING TORQUE WITH DIFFERENT SLOT OPENING WIDTH

Cogging torque can be defined as the negative derivative of the co-energy W with respect to the rotor position angle α, i.e.

$$T_{cog} = -\frac{\partial W}{\partial \alpha} \tag{3}$$

Since the energy stored in iron can be negligible, thus the magnetic co-energy stored in the machine can be taken as that stored in the air gap and PMs, i.e.

$$\begin{aligned} W &\approx W_{gap} + W_{PM} = \frac{1}{2\mu_0}\int_V B^2 dV \\ &= \frac{1}{2\mu_0}\int_V B_r^2(\theta)\left(\frac{h_m(\theta)}{h_m(\theta)+g(\theta,a)}\right)^2 dV \\ &= \frac{1}{2\mu_0}\int_V B_r^2(\theta) G^2(\theta,\alpha) dV \end{aligned} \tag{4}$$

A. Fourier Expansion of $B_r^2(\theta)$

When the magnetic poles distribute uniformly, the Fourier expansions of $B_r^2(\theta)$ can be expressed in following forms

$$B_r^2(\theta) = B_{r0} + \sum_{m=1}^{\infty} B_{rm}\cos 2mp\theta \tag{5}$$

B. Fourier Expansion of $G^2(\theta,\alpha)$

When the different slot opening width method proposed in this paper is adopted, the Fourier expansion of $G^2(\theta,\alpha)$ should be expanded at the interval $[-\pi, \pi]$ as follows.

$$G^2(\theta,\alpha)=G_0+\sum_{n=1}^{\infty}G_n\cos n(\theta+\alpha) \qquad (6)$$

When the slot number is odd, the Fourier expansion coefficients G_n is derived as

$$\begin{aligned}G_n&=\frac{2}{\pi}\Big(\sum_{i=1}^{z_s-1}\int_{\frac{\theta_a}{2}+(i-1)(\theta_b+\theta_c)}^{\frac{\theta_a}{2}+(i-1)(\theta_b+\theta_c)+\theta_c}g_s\cos n\theta d\theta+\int_{\pi-\frac{\theta_b}{2}}^{\pi}g_s\cos n\theta d\theta\Big)\Big)\\&=\frac{2g_s}{nz\pi}\{(-1)^{nz}\sin\frac{nz\theta_c}{2}+2\sin\frac{nz\theta_c}{2}\frac{\sin\frac{nz}{2}(z_s-1)(\theta_b+\theta_c)}{\sin\frac{nz}{2}(\theta_b+\theta_c)}\\&\cos[\frac{nz(\theta_a-2\theta_b-\theta_c)+nzz_s(\theta_b+\theta_c)}{2}]\}\end{aligned} \qquad (7)$$

where z_s is defined as $(z+1)/2$, g_s is defined as $(\frac{h_m}{h_m+g_{gap}})^2$.

When the slot number is even, the Fourier expansion coefficients G_n is derived as

$$\begin{aligned}G_n&=\frac{2}{\pi}\sum_{i=1}^{\frac{z}{2}}\int_{\frac{\theta_a}{2}+(i-1)(\theta_b+\theta_c)}^{\frac{\theta_a}{2}+(i-1)(\theta_b+\theta_c)+\theta_c}g_s\cos n\theta d\theta=\\&\frac{2g_s}{nz\pi}2\sin\frac{nz\theta_c}{2}\frac{\sin\frac{nz}{2}(\frac{z}{2})(\theta_b+\theta_c)}{\sin\frac{nz}{2}(\theta_b+\theta_c)}\cos\frac{nz(\theta_a-2\theta_b-\theta_c)+nz(\frac{z}{2}+1)(\theta_b+\theta_c)}{2}\end{aligned} \qquad (8)$$

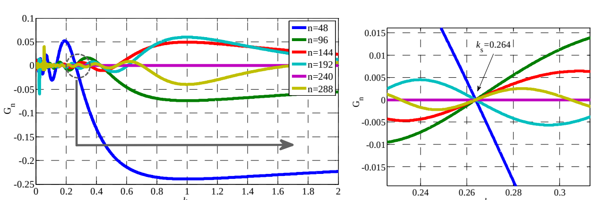
C. Analytical Model of Cogging Torque

Substituting (4)-(8) into (3), the analytical expression of cogging torque when adopting the proposed method can be derived as follows:

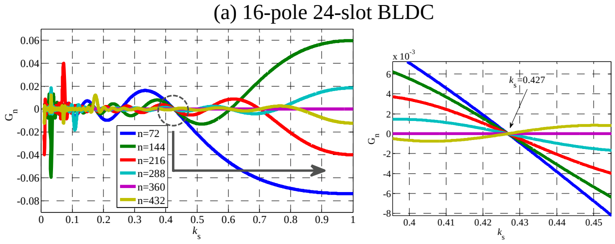
$$T_{cog}(\alpha)=\frac{\pi pL_{Fe}}{2\mu_0}(R_2^2-R_1^2)\sum_{n=1}^{\infty}nG_nB_{rn}\sin 2pn\alpha \qquad (9)$$

In (9), the expression of G_n is shown as (7) if the number of slots is odd, whereas it is shown as (8) if the number of slots is even, L_{Fe} is the stack length of armature, R_1 the outer radius of rotor yoke, R_2 the inner radius of stator.

D. Determining the Slot Opening width by Analytical Method



(a) 16-pole 24-slot BLDC



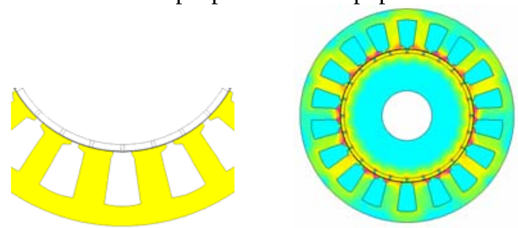
(b) 24-pole 18-slot BLDC

Fig. 3 G_n calculated by analytical model for 16-pole, 24-slot PMSM

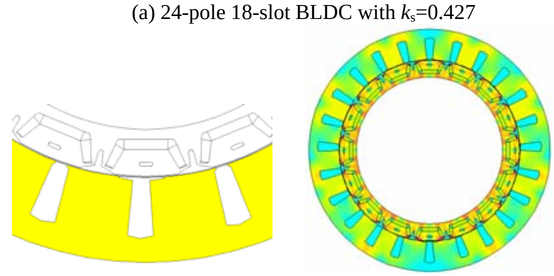
According to (7) and (8), G_n can be calculated analytically with different k_s. Fig.3 shows the value of G_n corresponding to different k_s for the prototype machines.

IV. Validation by FEM

In order to verify the correctness and effectiveness of the proposed method, FEA is carried out to calculate the cogging torque corresponding to two different k_s, respectively, i.e. k_s=1, k_s determined by analytical method. Fig.5 show the magnetic field distribution of the prototype machines and Fig.6 shows the cogging torque results calculated of the two prototype machines which strongly validate the method proposed in this paper.

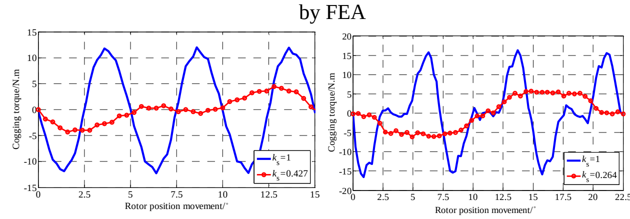


(a) 24-pole 18-slot BLDC with k_s=0.427



(b) 16-pole 24-slot IPMSM with k_s=0.264

Fig. 5 Slot area with different k_s and magnetic field distribution calculated by FEA



(a) 24-pole, 18-slot BLDC (b) 16-pole, 24-slot IPMSM

Fig. 6 Cogging torque calculated by FEM

V. References

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